A new post-quantum digital signature scheme

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SIAM-AG TU/e
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joint work (in progress) with Ross Bowden

Concept: Let's take the Q out of SQISign

public, secret, commitment, challenge, response

1

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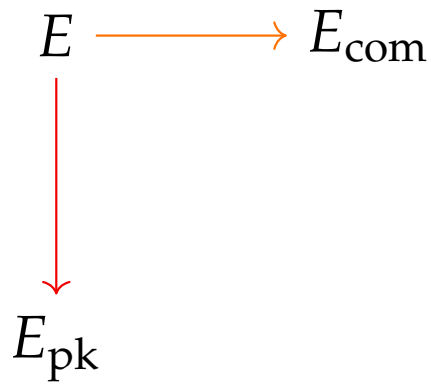

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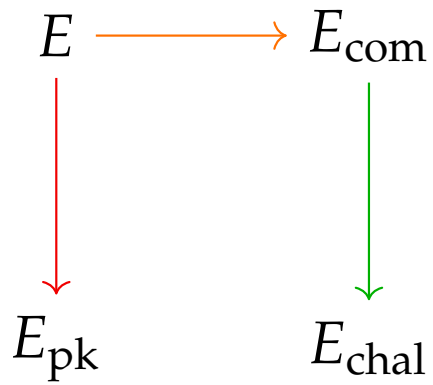

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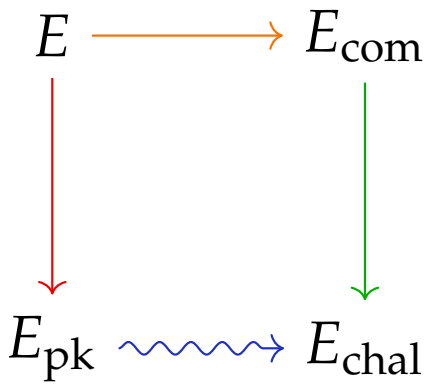

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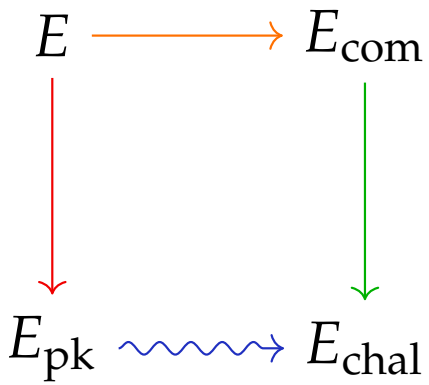

public, secret, commitment, challenge, response

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  - Computing $\mathrm{End}(E_{\text{chal}})$
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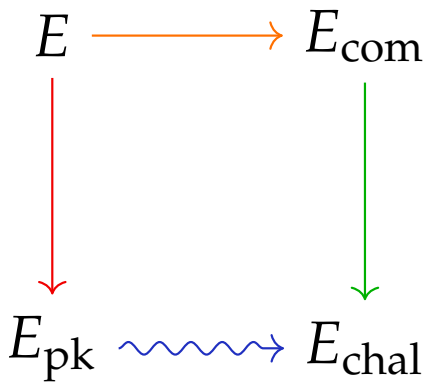

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- Idea: replace
  - Computing $\mathrm{End}(E_{\text{chal}}) \rightsquigarrow$ basis change on $T_2(E_{\text{pk}})$
  - KLPT $\rightsquigarrow$ once-for-each-E_{pk} computation[1]

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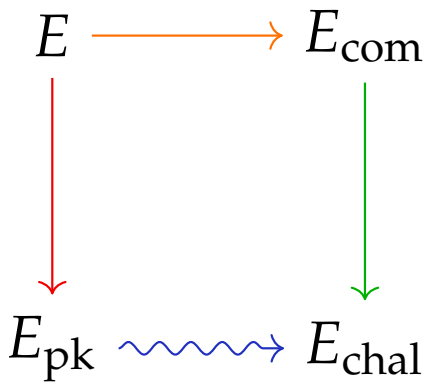

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How?!

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How: Bruhat-Tits trees and isogeny graphs

Quaternion Algebra graphs

Nodes:
Maximal orders / $\sim$

Edges:
Ideals of norm 2 / $\sim$

Supersingular isogeny graphs

Nodes:
j-invariants

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Quotients of Bruhat-Tits trees

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Quotients of Bruhat-Tits trees

Nodes:

$$\left\{ \begin{pmatrix} 2^n & 0 \\ r & 2^m \end{pmatrix} : 0 \leq r < 2^m \right\} / \sim$$

Edges:

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

act on nodes via multiplication.

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(Quotients of)
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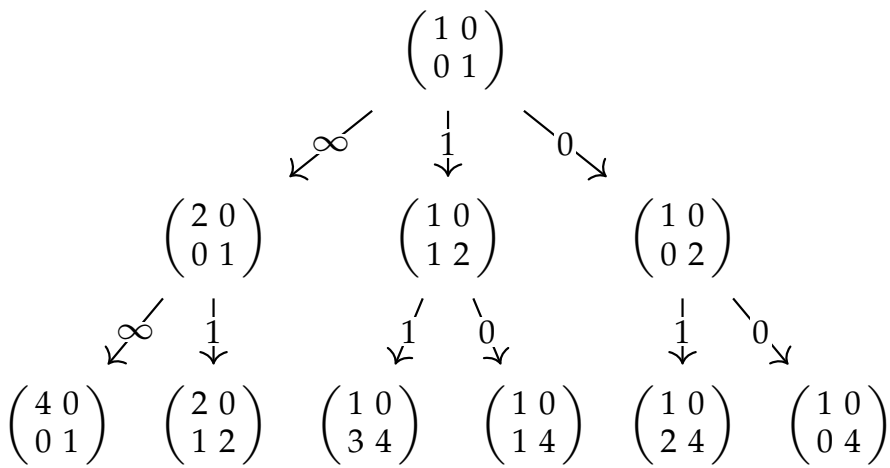

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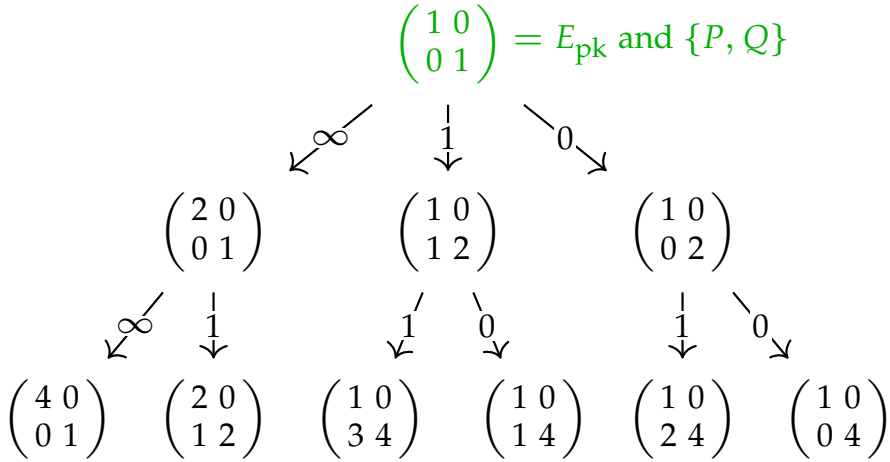

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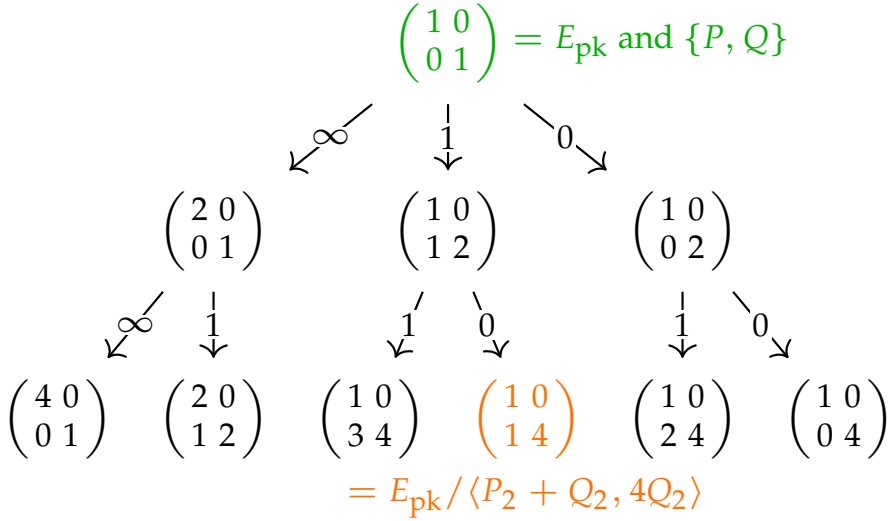

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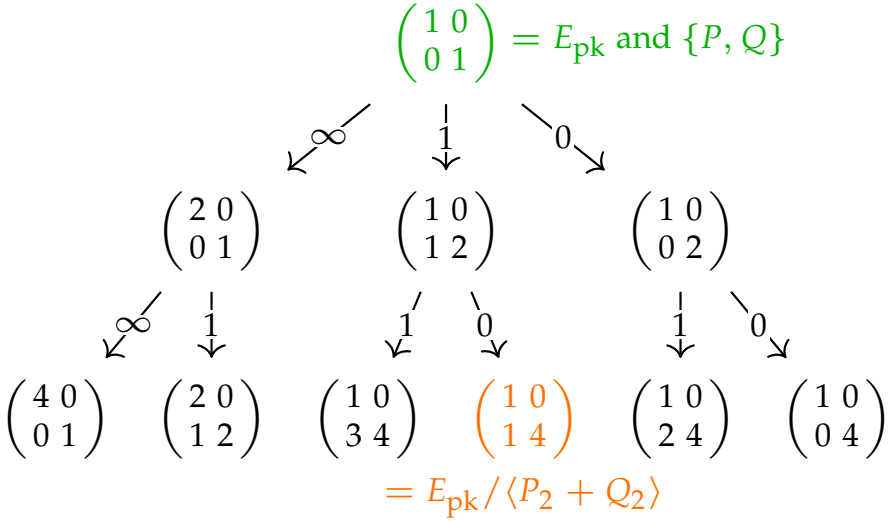
3. A matrix node M at depth d is

$$E_{\text{pk}}/\langle (P_d, Q_d) \cdot M \rangle.$$



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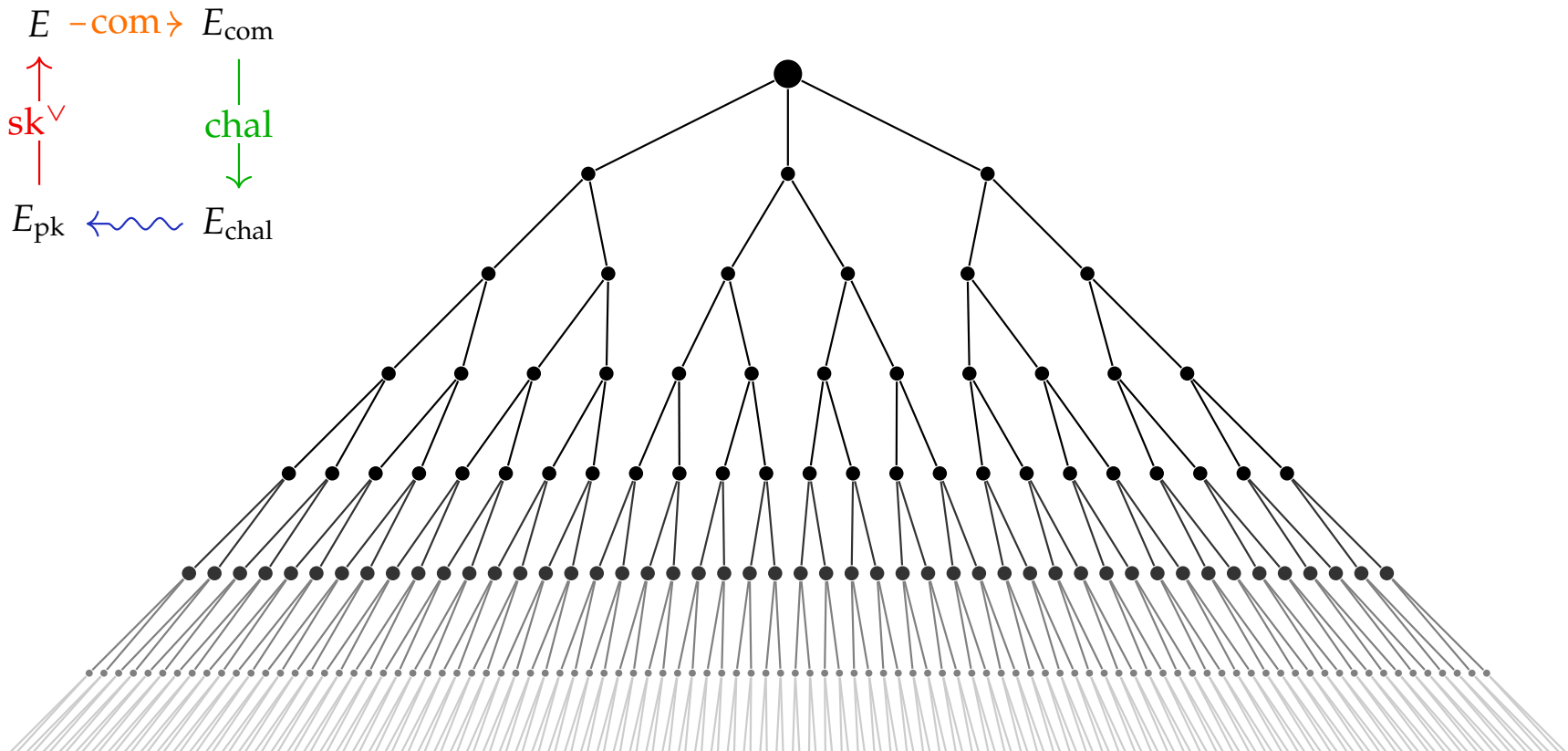

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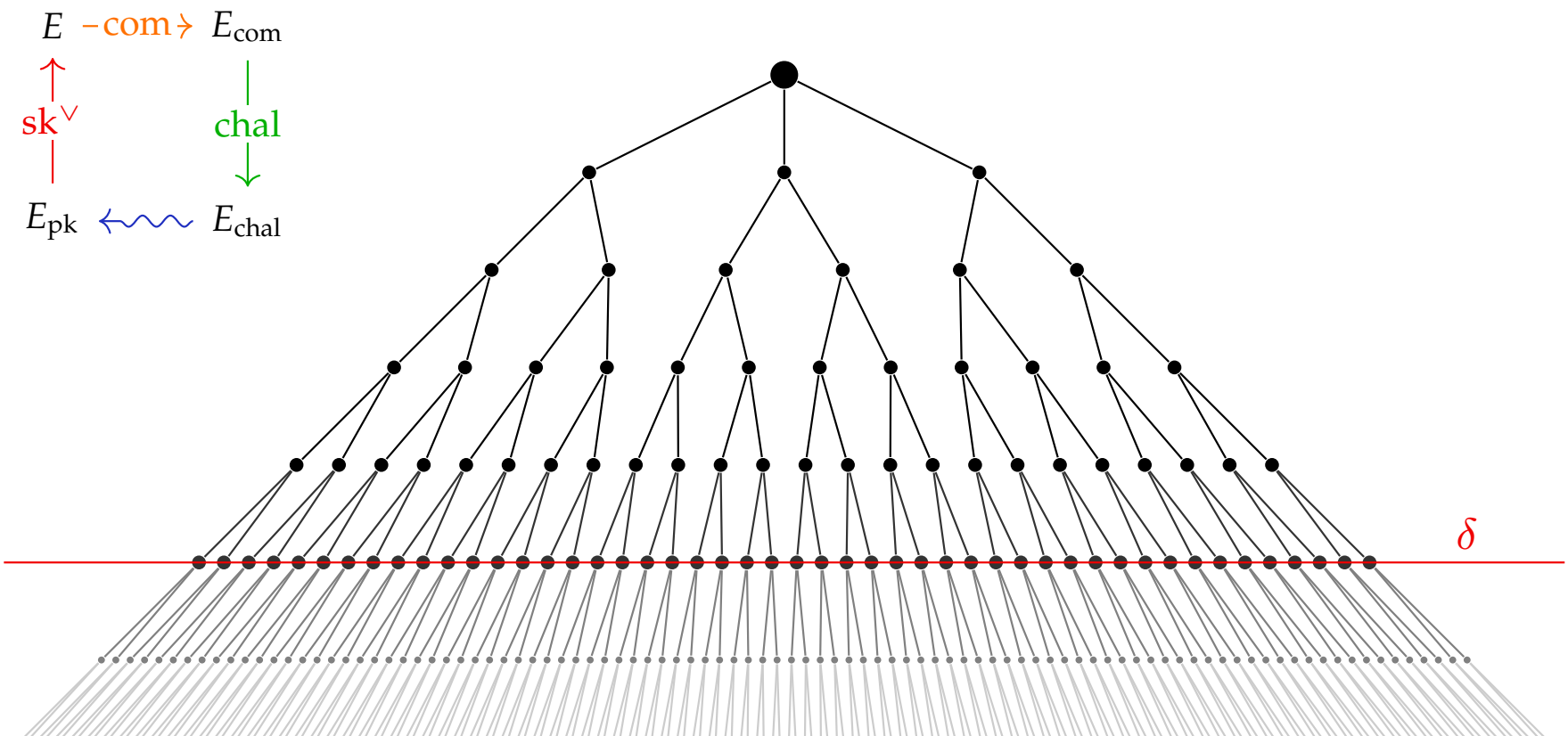

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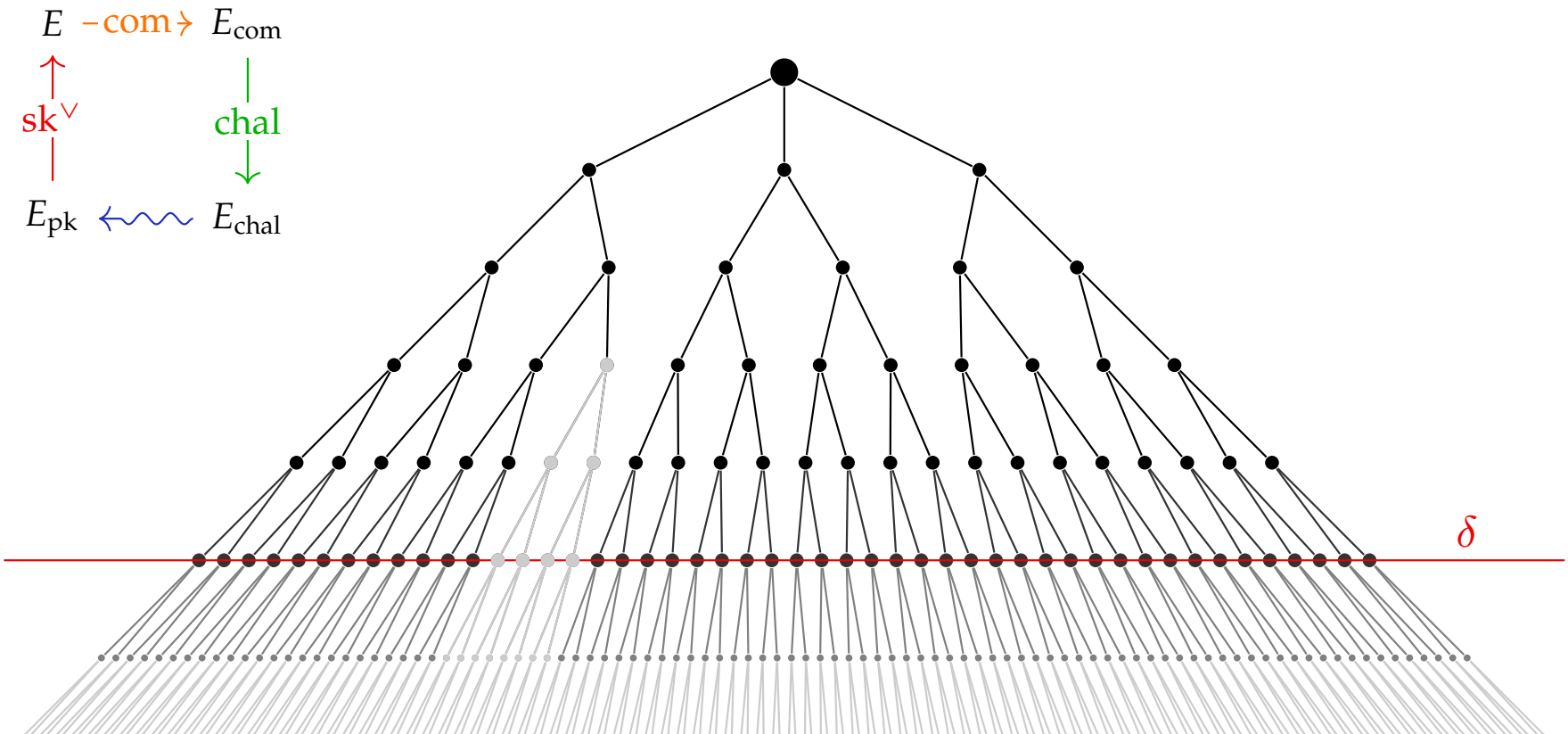

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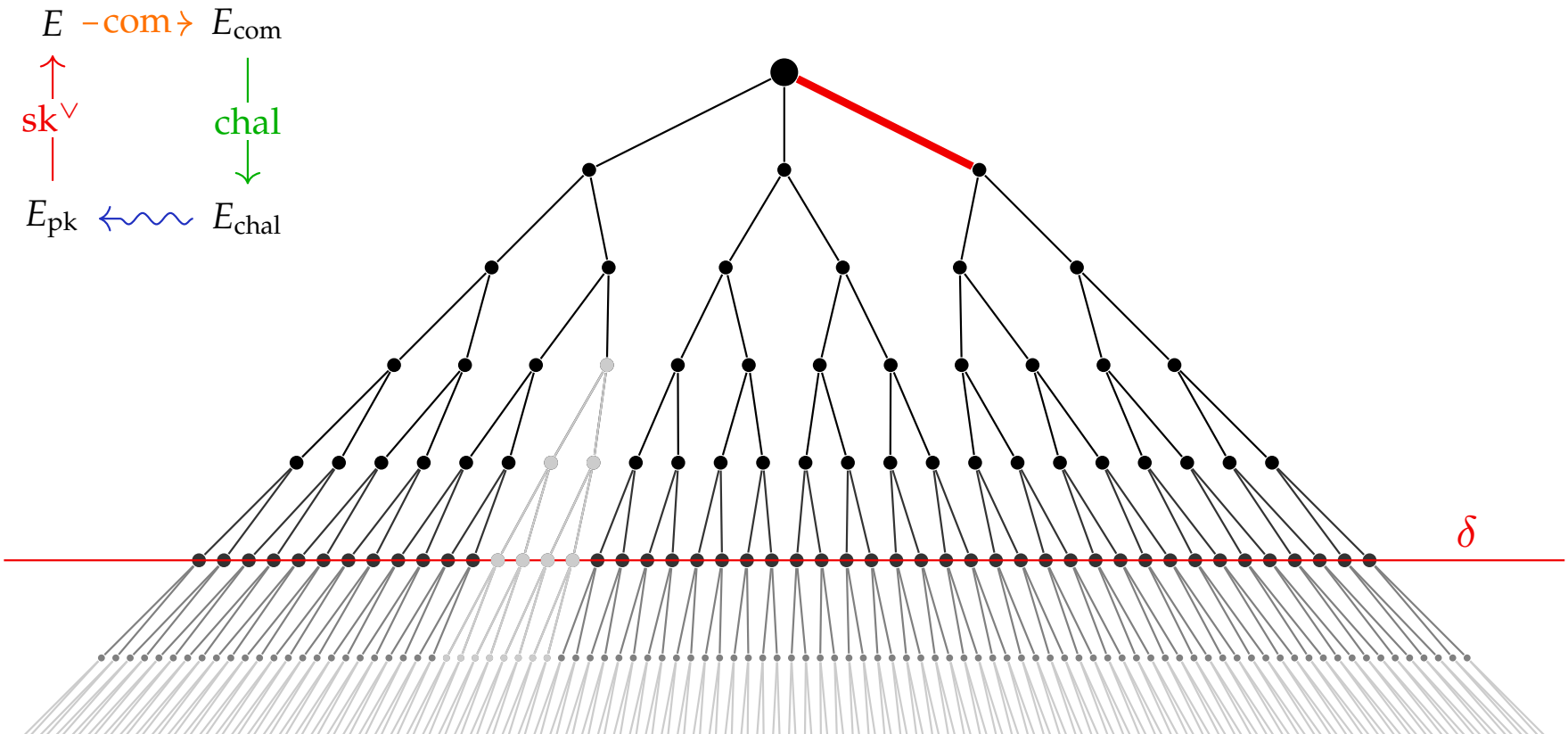

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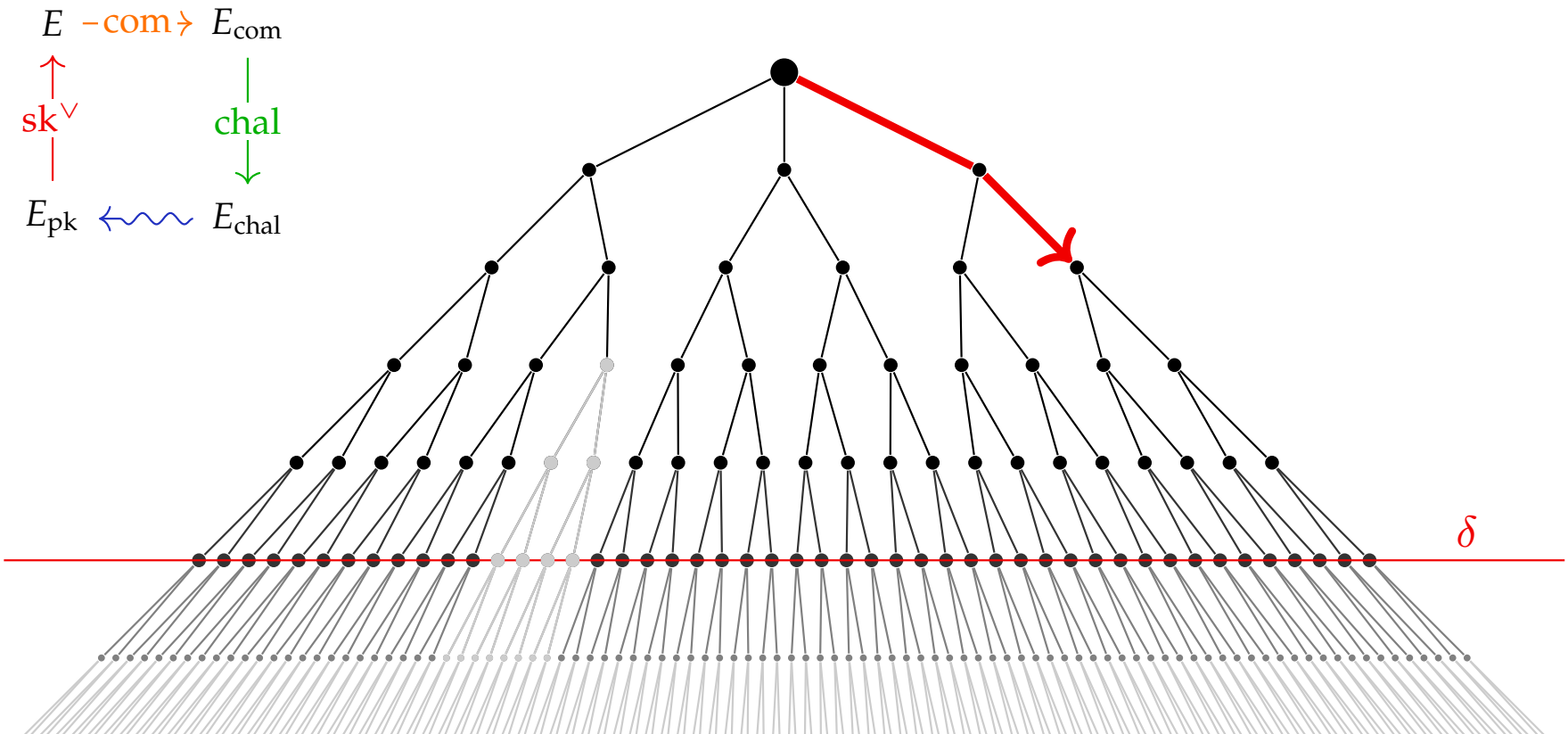

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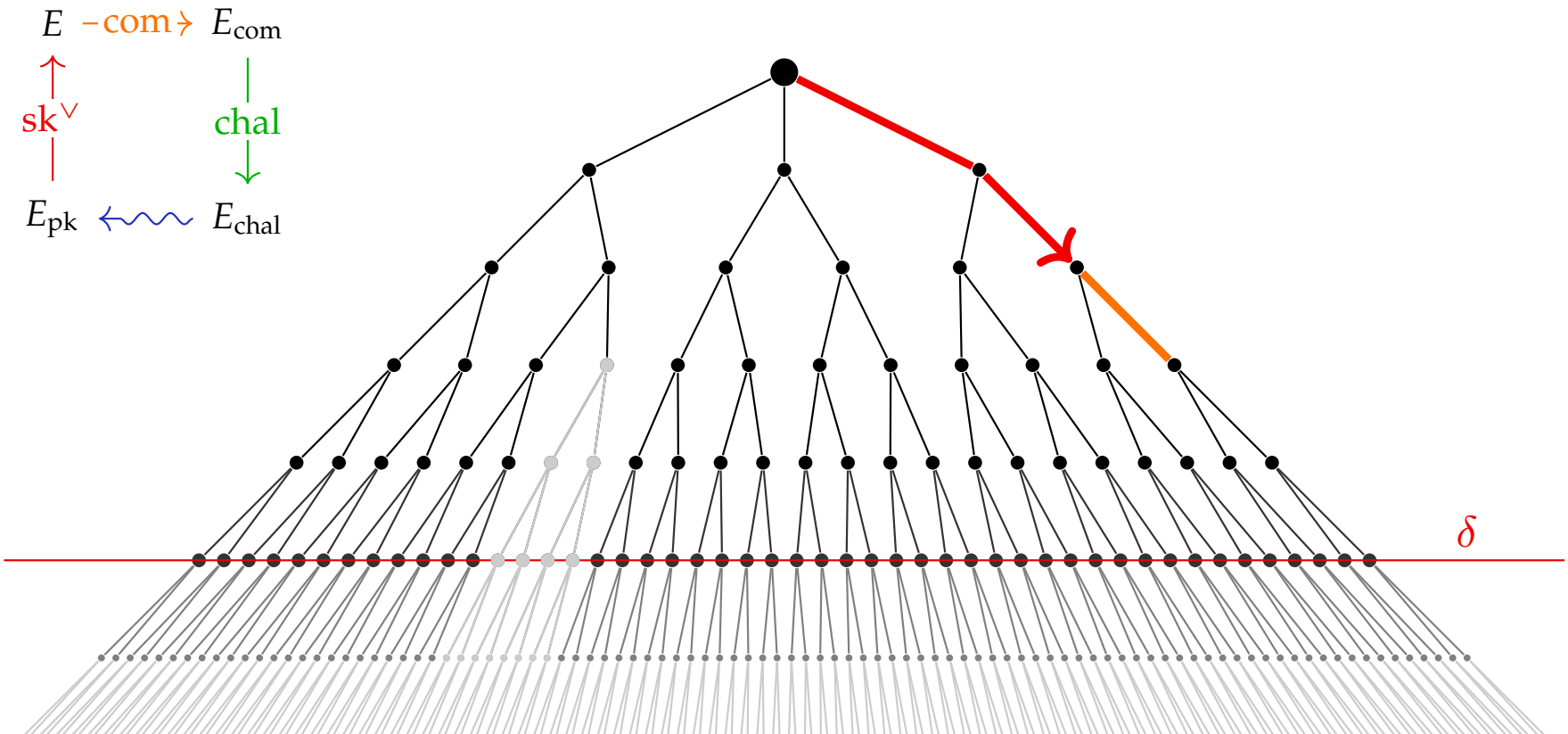

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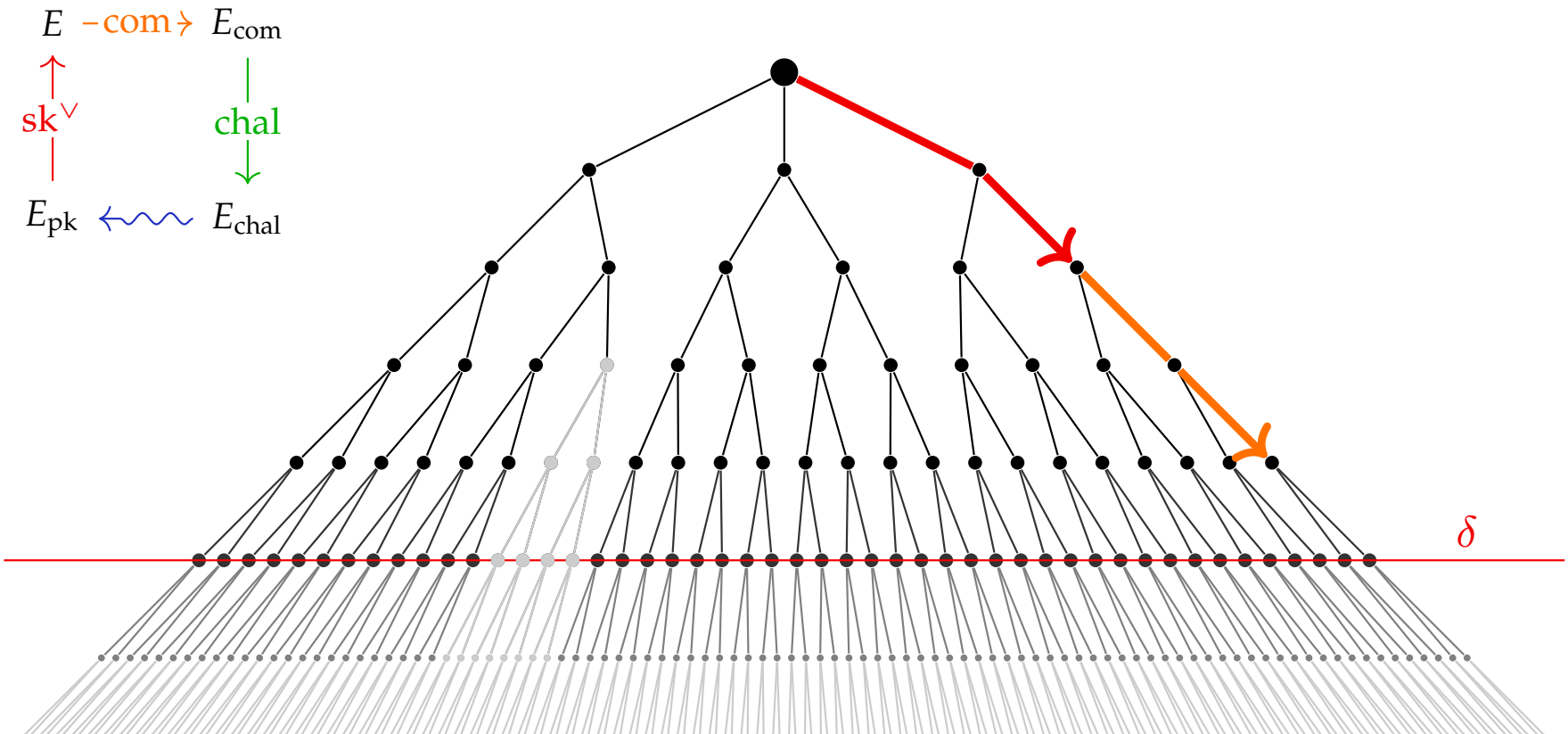

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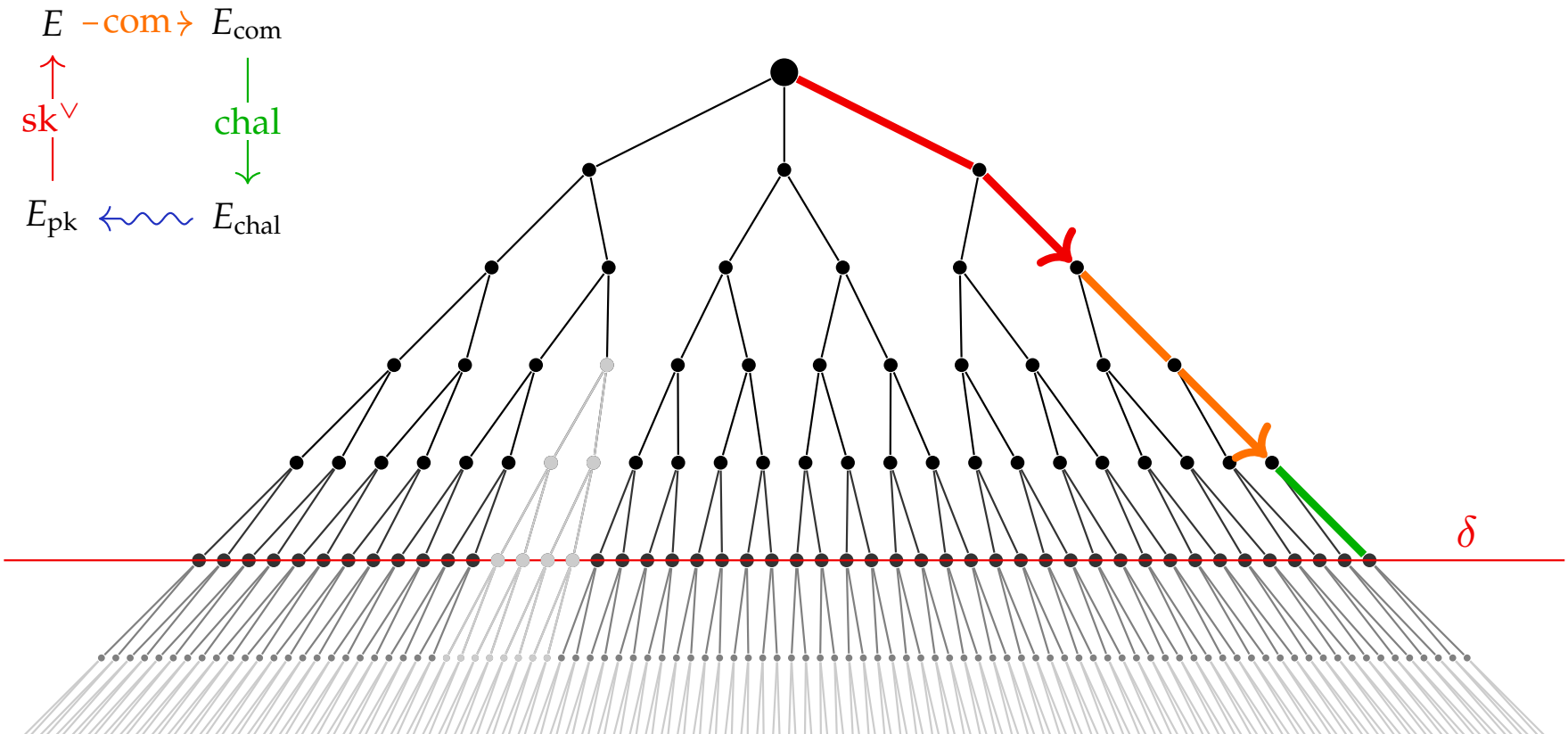

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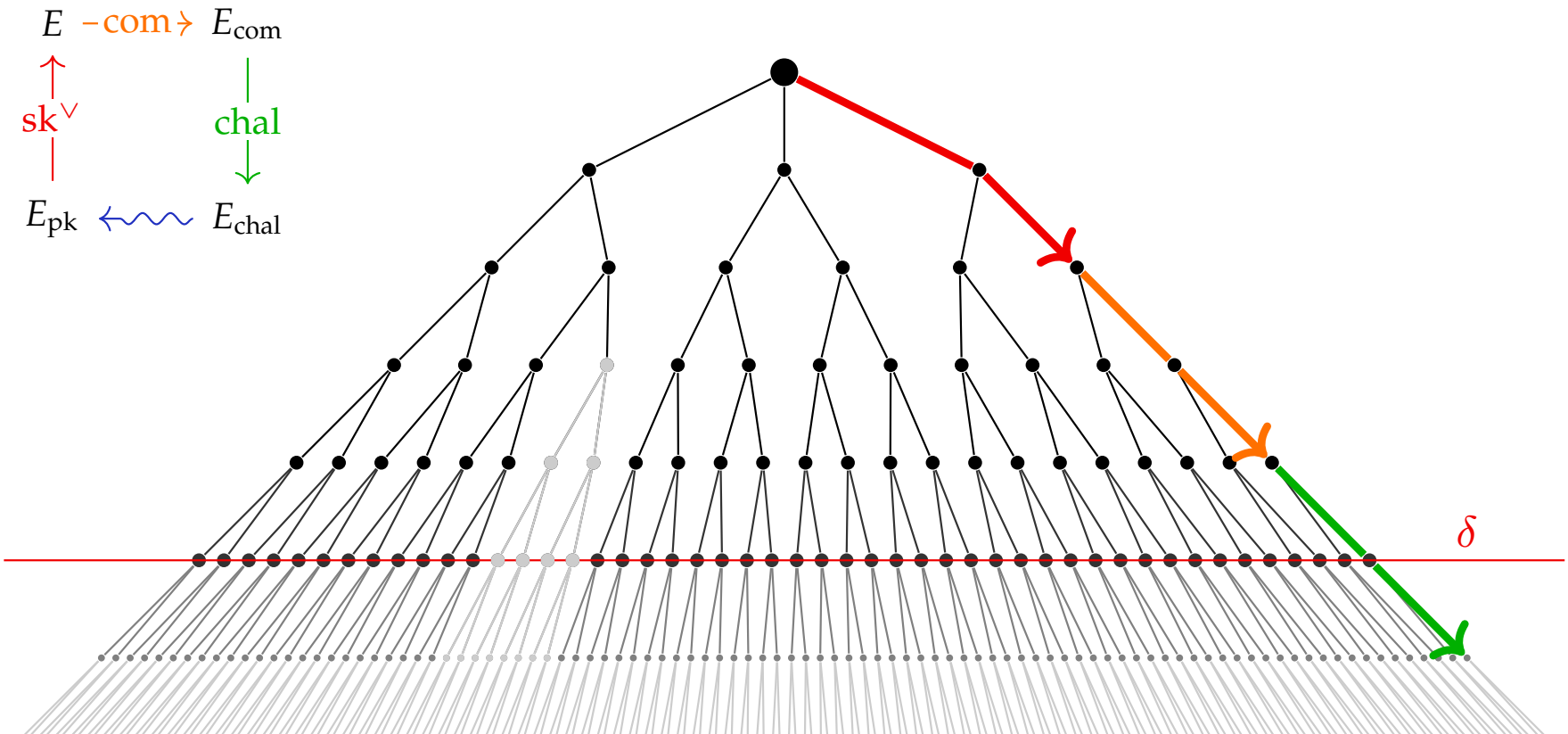

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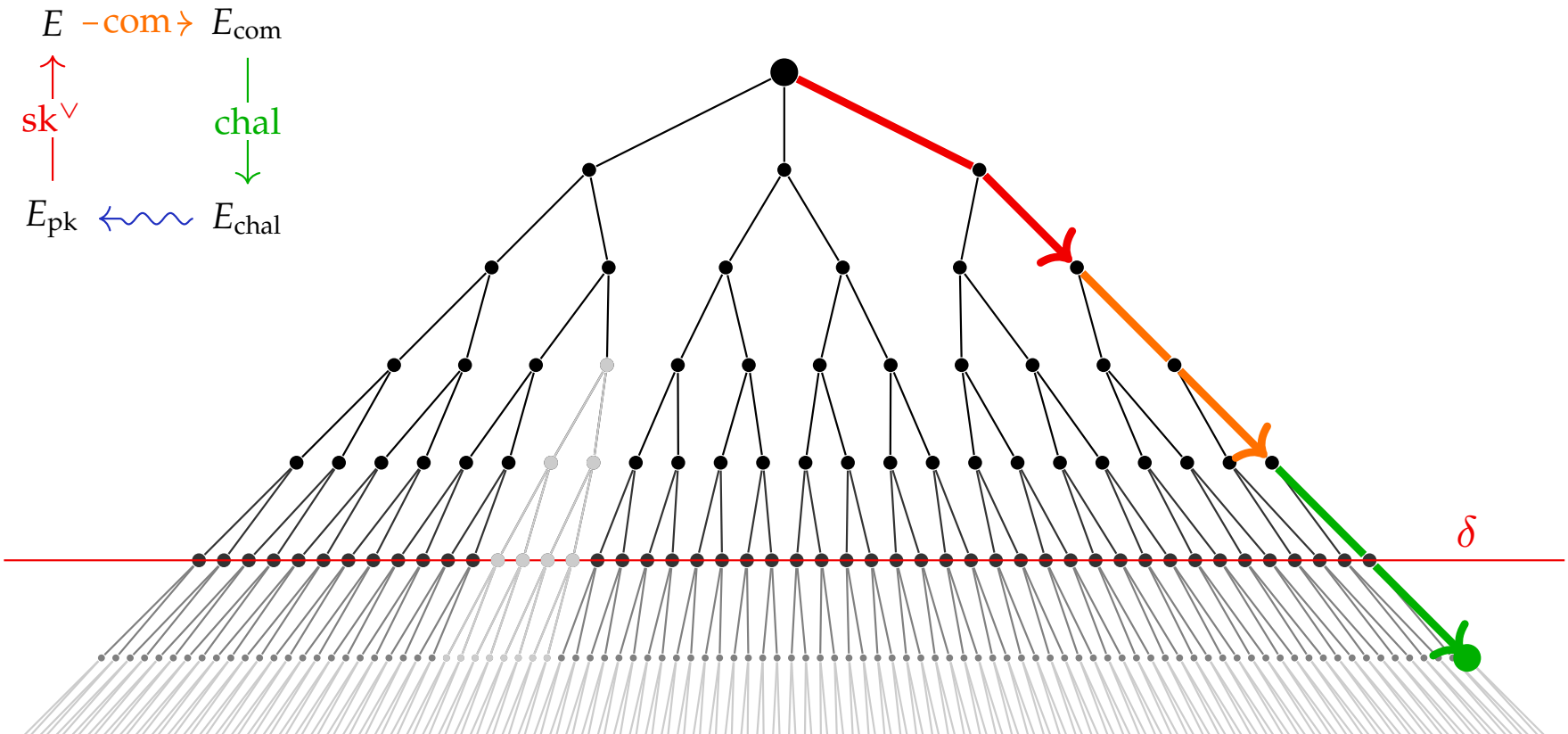

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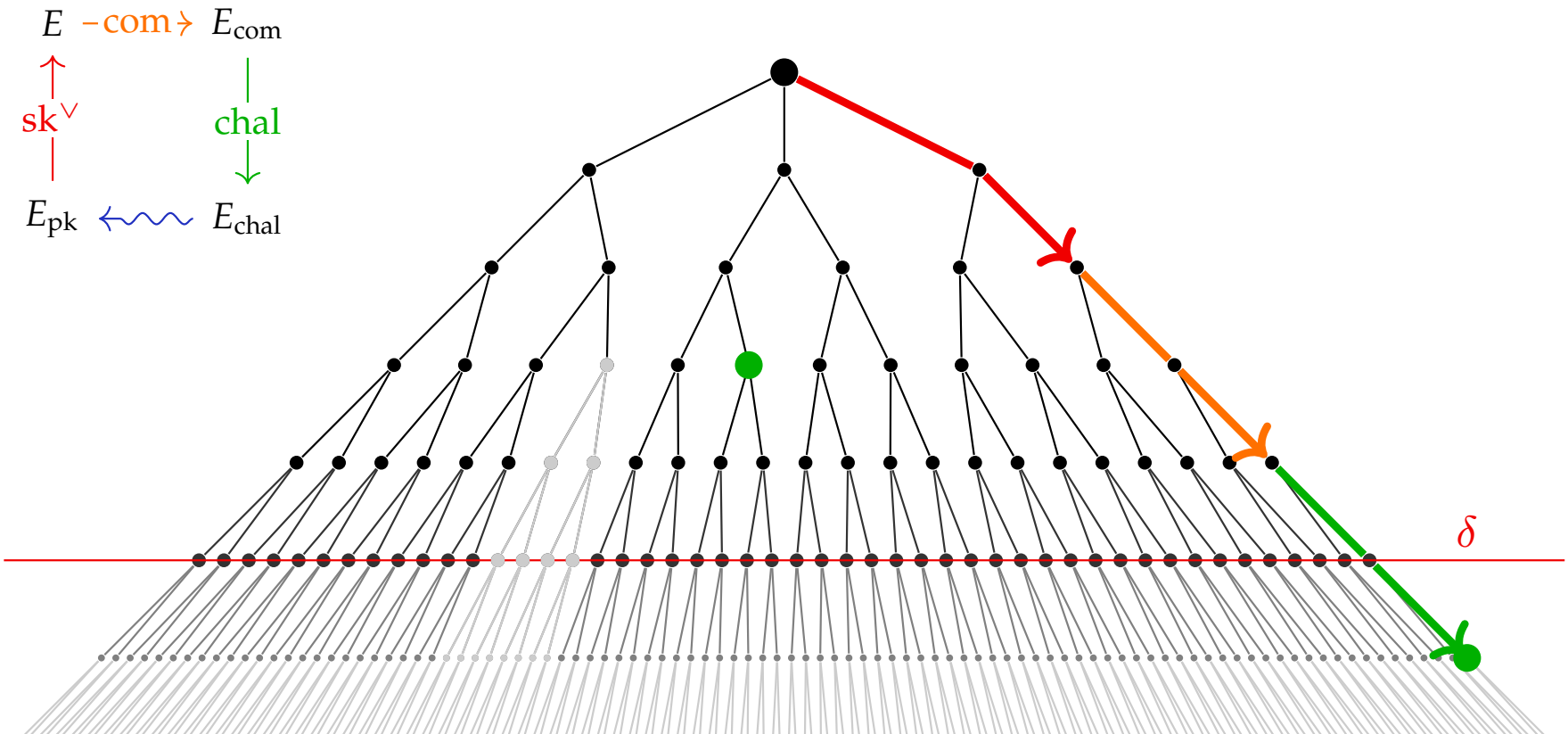

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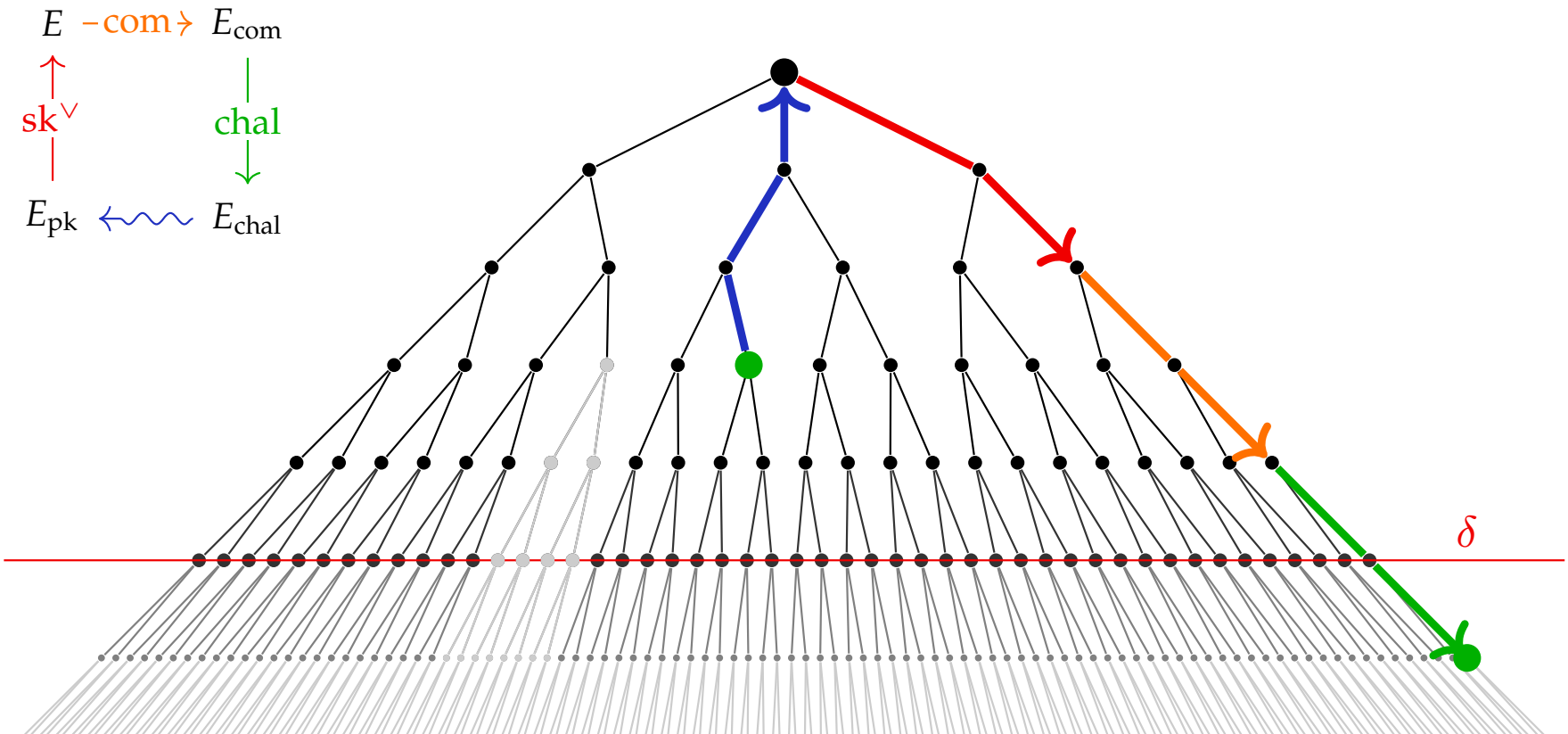

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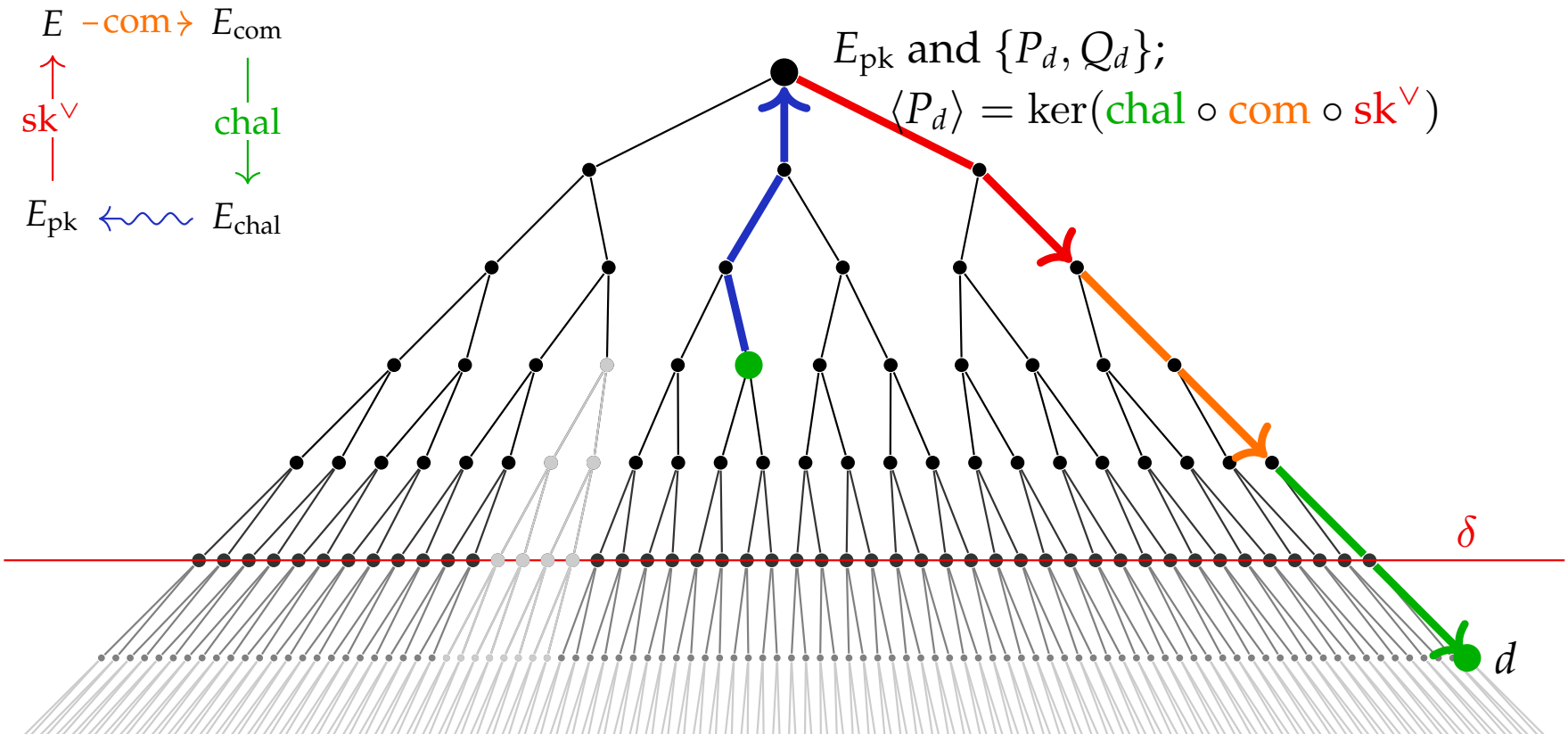

The dream: A Bruhat-Tits identification protocol

Setup:

1. Starting at $j(E_0) = 1728$, compute a chain of 2-isogenies sk; publish codomain E_{pk} as public key.

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3. Compute, for these choices, the matrix node M of depth $< \delta$ equivalent to $\begin{pmatrix} 1 & 0 \\ 0 & 2^d \end{pmatrix}$ in the quotient of the Bruhat-Tits tree with root E_{pk} and basis $\{P_d, Q_d\}$ of $T_2(E_{\text{pk}})$ at precision 2^d.

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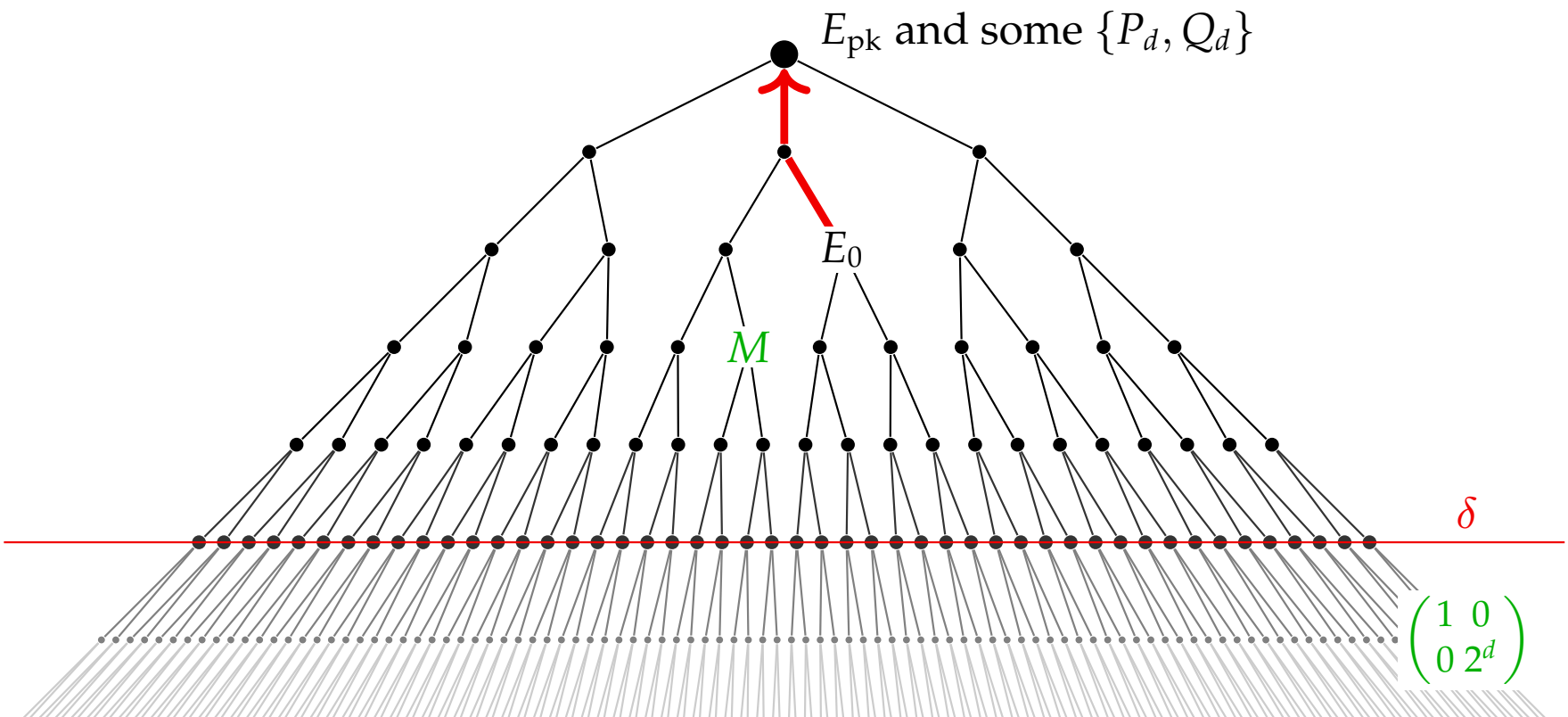

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$$\text{com} : E_0 \longrightarrow E_{\text{com}};$$

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Response:

1. Compute the base change to basis $\{P'_d, Q'_d\}$ such that
$$\langle P'_d \rangle = \ker(\text{chal} \circ \text{esk} \circ \text{sk}^\vee).$$
2. Using M, read off a shorter path back to the root, reveal the corresponding 2-isogeny chain.

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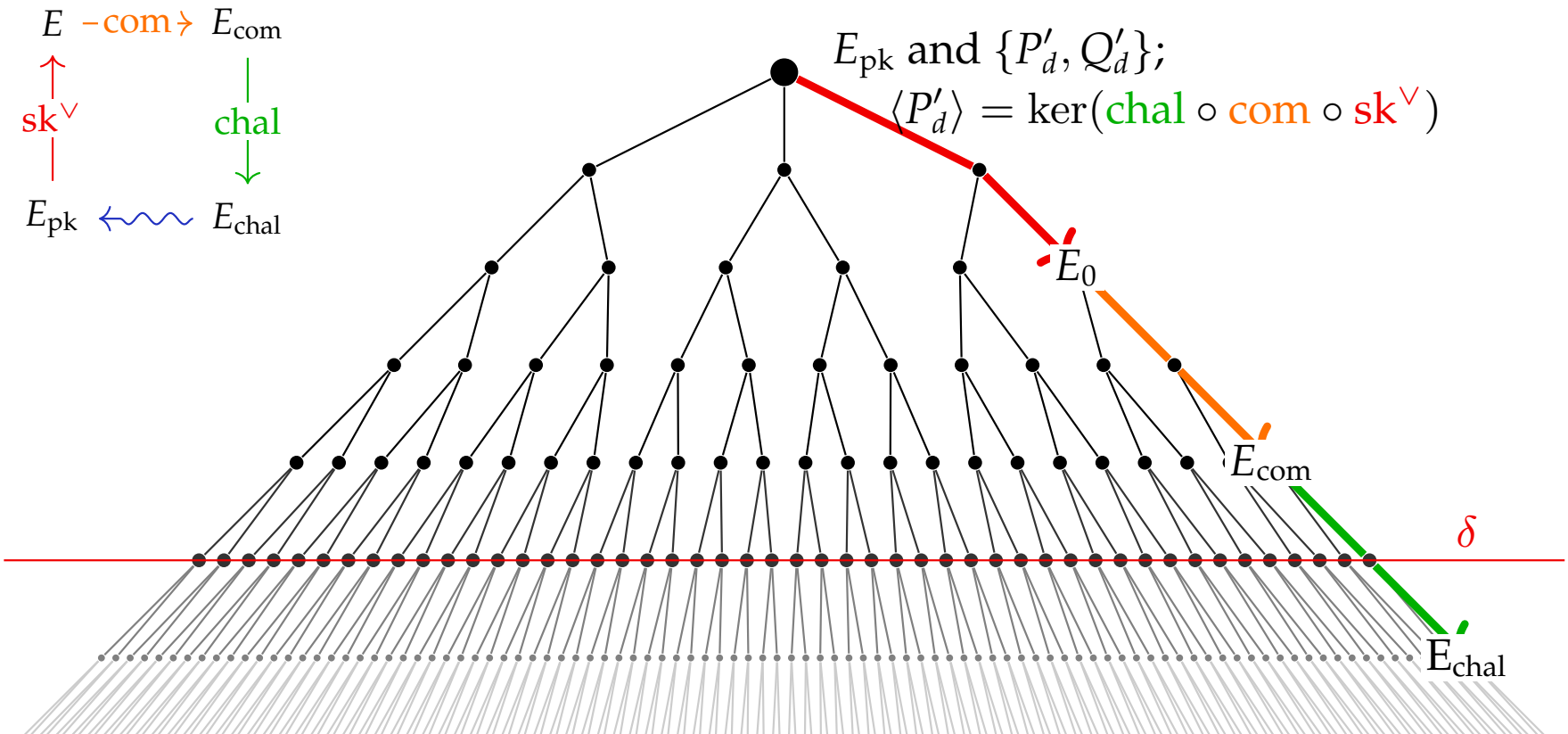

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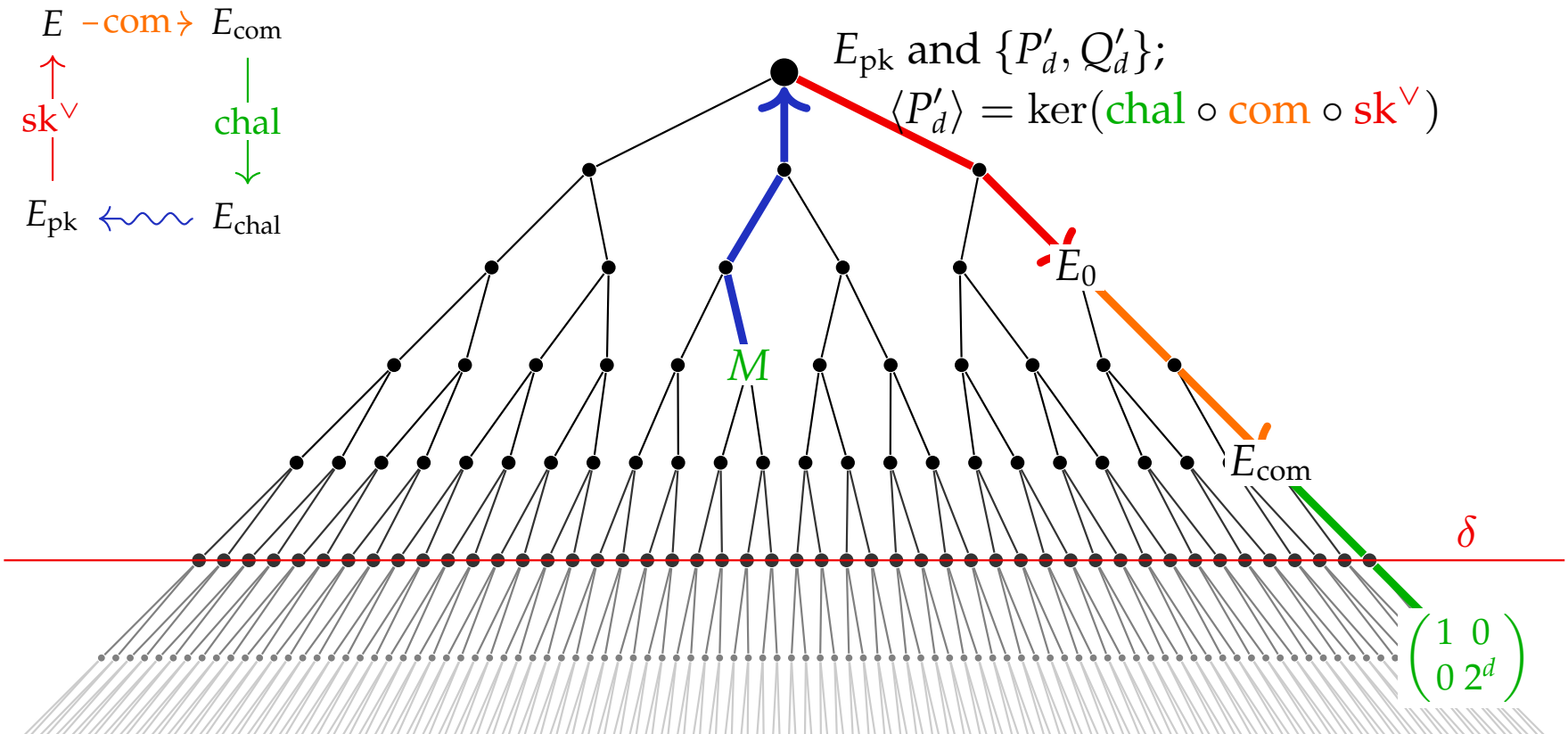

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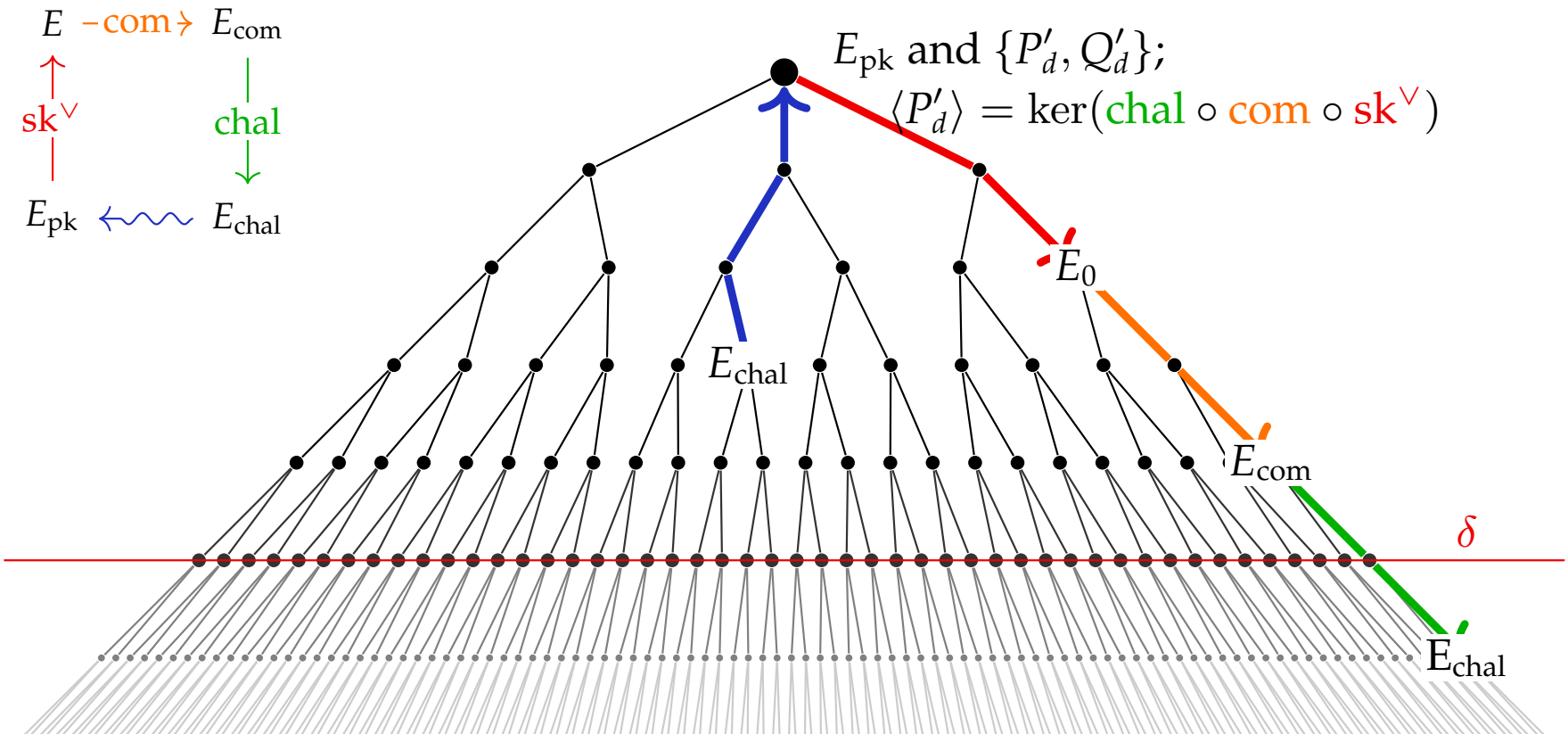

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- We can't yet compute $M \sim \begin{pmatrix} 1 & 0 \\ 0 & 2^d \end{pmatrix}$.
- Only existing algorithm: Franc-Masdeu.
- Adapting F-M leads to:

Open question:

Find the shortest vector of

$$\begin{pmatrix} N \\ 2^n I \end{pmatrix} \in \mathrm{Mat}_{4\times 8}(\mathbb{Z}[x])$$

with respect to length

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Questions? Answers?

References

AILMS Amorós, Iezzi, Lauter, Martindale, and Sotáková *Explicit connections between supersingular isogeny graphs and Bruhat-Tits trees*
`https://ia.cr/2021/372`

DKLPW De Feo, Kohel, Leroux, Petit, and Wesolowski, *SQISign: compact post-quantum signatures from quaternions and isogenies*
`https://ia.cr/2020/1240`

FM Franc and Masdeu, *Computing fundamental domains for the Bruhat-Tits tree for $GL_2(\mathbb{Q}_p)$, p-adic automorphic forms, and the canonical embedding of Shimura curves*
`https://arxiv.org/abs/1201.0356`
`https://github.com/mmasdeu/btquotients`